Alternative FEL Concepts
short-bunch compact FEL

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Outline

1. Compact FEL
2. Chirped-pulse FEL
3. Laser-accelerator driven FEL
**Small ≈ Economic ≈ Stable**

**outline**

1. Reducing beam energy → THz FEL → reducing both accelerator/undulator length

2. Increasing acceleration gradient → laser-driven particle accelerator → shortening accelerator with fixed beam energy

3. Increasing startup power → particle bunching → shortening undulator length to reach saturation power

4. Reducing beam energy and undulator period → reducing both accelerator/undulator lengths for XFEL → quantum FEL vs. classic FEL
FELO Small Signal Gain – lasing requirement: $\text{gain} > \text{loss}$

$$G - 1 = 2\pi \frac{a_u^2 e L_u^2 N_u}{\varepsilon_0 \gamma^3 m_0 c_0^3} J_e \times F(\xi) \times g(\theta) \propto \frac{1}{\gamma^3}$$

Factor $\sim 1$ Gain function

Shortening an FEL $\rightarrow$ reducing $\gamma$ (beam energy) $\rightarrow$ shortening accelerator

Recall the FEL Synchronism Condition

$$\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2}$$

undulator period: $\lambda_u$

Undulator parameter

$$a_w = \frac{e B_{u,\text{rms}}}{m_0 c_0 k_w} = \frac{e A_{u,\text{rms}}}{m_0 c_0} = 0.093 B_{u,\text{rms}} \text{(kG)} \times \lambda_u \text{(cm)}$$

Reducing $\gamma$ $\rightarrow$ THz FEL
The Stanford $300k, 3.5$ THz Sub-compact FIRFEL

The RF/FIRFEL System Configuration

The Stanford Sub-compact FIRFEL
SASE FEL

Goal: shortening a FEL for a given radiation wavelength $\lambda_r$

Shortening an FEL = Shortening Accelerator + Shortening Undulator
Shortening Accelerator for same $\gamma$ (1) – Laser plasma accelerator

increasing acceleration gradient $\implies$ laser driven particle acceleration $\implies$ shortening accelerator for same $\gamma$

Laser/plasma wakefield Accelerator

Short laser pulse

$\tau_{\text{laser}} < \frac{\lambda_p}{c}$

For $r \sim \lambda_p \sim 20 \ \mu m$

$N \sim 5 \times 10^{17}/\text{c.c.}$

Gauss Law $\implies E_a \sim \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{rNe}{3\varepsilon_0}$

$\sim 100 \text{ GV/m}$

km accelerator for XFEL reduces to m long!

http://www.youtube.com/watch?v=MINxgmPVF6U
Shortening Accelerator for same $\gamma$ (2) – laser plasma accelerator

Synchrotron radiation generated using an electron beam from a laser-driven accelerator opens the possibility of building an X-ray free-electron laser hundreds of times smaller than conventional facilities currently under construction.

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Synchrotron radiation sources have become an indispensable tool in a wide range of disciplines, including physics, biology, materials science, chemistry and medicine. The reason they are so useful is the high intensity of X-rays they produce — generated when the path of a beam of electrons moving at relativistic speeds is bent by a periodic magnetic field — in comparison with other X-ray sources. Such utility is expected to grow still further with the development of X-ray free-electron lasers.

Shortening Accelerator for same $\gamma$ (3) – laser plasma accelerator

TABLE I. Parameters of different scenarios for realizing free-electron lasers from laser-driven plasma-based accelerators in the XUV and x-ray domains with the corresponding FEL quantities by combining them with conventional undulators. The scenarios X-FEL 1 and 2 highlight the possibility to produce strong FEL radiation from a laser-plasma accelerator. The most challenging condition is the relative energy spread of the electron beam, which has to be decreased to attain the x-ray part of the spectrum. In addition, high-quality electron beams at the GeV level are required [1 GeV electron beams have already been generated (Leemans et al., 2006) but not with the same quality, i.e., energy spread and stability, as at the 100 MeV level]. The notion of slice energy spread and slice

<table>
<thead>
<tr>
<th>Pulse energy</th>
<th>50 $\mu$J</th>
<th>25 $\mu$J</th>
</tr>
</thead>
<tbody>
<tr>
<td>rms pulse duration</td>
<td>2 fs</td>
<td>2 fs</td>
</tr>
</tbody>
</table>

Typical SASE FEL Buildup Curve

\[ P_s = \rho P_B \]

saturation

\[ P = P_\omega \exp \left( \frac{z}{L_g} \right) \]

\( P_\omega \): Startup power

\[ L_g = \frac{\lambda_u}{4\pi \sqrt{3} \rho} \]

Gain length

\[ \rho = 1.78 \times 10^{-5} \frac{A_u^{2/3}}{\gamma} \times \lambda_u^{2/3} [cm] n_e^{1/3} [cm^{-3}] \]

Pierce parameter

\[ A_u = a_u \] undulator parameter for helical undulator

Beam Power

\[ P_B = IV_B \]
Shortening Undulator: large startup $P_\omega$

\[ P_s = \rho P_B \quad \text{Saturation power} \]

\[ P(z) = P_\omega \exp(z/L_g) \]

\[ \begin{aligned} 
\{ \text{Undulator length} \quad L_u &= L_g \ln(\rho P_B / P_\omega) 
\} 
\end{aligned} \]

To shorten $z$ with constant saturation power $P_s$, large startup power $P_\omega \uparrow$ and short gain length $L_g \downarrow$ are preferred.

The startup power (mostly from spontaneous radiation) is related to the spectral content of the electron distribution.
Spontaneous Radiation from many electrons

\[
\left( \frac{dW}{d\omega} \right)_N = \left( \frac{dW}{d\omega} \right)_1 \left| \sum_{i=1}^{N} e^{-j\phi_i} \right|^2
\]

Assume no energy exchange between electrons

\[
\left( \frac{dW}{d\omega} \right)_1
\]
Spectral energy radiated by a single particle

\[\phi_i(t, r)\] Radiation phase of \(i^{th}\) particle

\[N: \text{number of electrons}\]

\[
\left| \sum_{i=1}^{N} e^{-j\phi_i} \right|^2 = \begin{cases} 
\sim N & \text{for random phase } \phi_i \\
N^2 & \text{for a constant phase } \phi_i = \phi_0 \end{cases}
\]

\[\text{superradiance}\]
Bunching Factor

\[ |M| = \left| \sum_{i=1}^{N} e^{-j\phi_i} \right| / N \quad 0 \leq |M| \leq 1 \]

In the limit of continuous distribution with a temporal distribution function \( f(t) \), where \( \int_{-\infty}^{\infty} f(t)dt = 1 \), the bunching factor becomes

\[ |M(\omega)| \rightarrow \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \equiv \text{Fourier Transform of } f(t) \]

\[ |M(\omega_n)| = 1 \quad \text{means perfect bunching at frequency } \omega_n, \text{ yielding the maximum enhancement factor of } N \text{ for radiation} \]
Incoherent Radiation

\[ \sigma_z \gg \lambda_r, r_i \text{ is random} \]

Total Spectral Energy

\[
\frac{dW}{d\omega}_{inc,N} = N \left( \frac{dW}{d\omega} \right)_1 \propto N \quad N: \text{number of electrons}
\]

Radiation spectral energy of a single electron

\[
\left( \frac{dW}{d\omega} \right)_1
\]
Coherent Spontaneous/Synchrotron (CSR) Radiation

\[ \sigma_z, \tau_b \]

\[ \lambda_r \]

\[ \sigma_z \ll \lambda_r \]

\[ \sum_{i=1}^{N} e^{-j\phi_i} \approx N^2 \]

\[ \phi_i \sim \text{constant} \]

\[ z, t \]

Spectral Energy

\[ \left( \frac{dW}{d\omega} \right)_{SR,N_b} = N_b^2 \left( \frac{dW}{d\omega} \right)_1 M_b^2(\omega) \]

\( M_b(\omega) \): Fourier transform of the bunch shape

\( N_b \): number of electrons in the bunch

* For 1 nC in 10 ps, \( N_b = 6.25 \times 10^9 \)!

For Gaussian bunch

\[ f(t) = \exp \left( -\frac{t^2}{2\tau_b^2} \right) \]

\[ \Rightarrow \quad M_b(\omega) = \exp \left( -\frac{\omega^2 \tau_b^2}{2} \right)^{\omega\tau_b} \]
CSR from Periodic Bunches (comb-like pulses)

Spectral Energy

\[
\left( \frac{dW}{d\omega} \right)_{SR} = \left( \frac{dW}{d\omega} \right)_1 N_b^2 M_b^2(\omega) \times N_p^2 M_p^2(\omega)
\]

- \( M_b(\omega) \): Fourier transform of the bunch shape
- \( N_b \): number of bunched electrons

\[
M_p(\omega) = \frac{\sin(N_p \pi \omega / \omega_b)}{N_p \sin(\pi \omega / \omega_b)}
\]

Coherent sum of \( N_p \) bunches with bunching freq. \( \omega_b \)
Desktop Narrow-line MW THz Free-electron Laser

THz-pulse-train laser

Solenoid

12 cm

50 cm

Photocathode gun

THz DFG
(Nonlinear optical material)

Narrow-line THz wave

Single-pass undulator

50~100 cm

10 MW power at THz

Maintaining

\[ \lambda_r = \frac{(1 + a_u^2) \lambda_u}{2 \gamma^2} \]

bunching factor = 0.2
bunching factor = 0.1
bunching factor = 0.01

> 14 MW

20% linear taper
Bunching Frequency Multiplication

- Macropulse
- Compressed macropulse

Bunching frequency is multiplied
Pulse compression schemes

Magnetic Chicane compression

Problems: Coherent synchrotron radiation, energy spread, emittance growth etc.

Accelerator velocity bunching

Problems: Coherent synchrotron radiation, energy spread, emittance growth etc.
A soft x-ray FEL with 10-time reduced size

Bunching wavelength
1\~1.5 \, \mu m \, (3\~5 \, \text{fs})

Chicane bunching compression
(bunching wavelength / 3\~5)

$\sim 10 \, \text{MeV}$

Velocity bunching
(bunching wavelength / 10\~15)

Nd laser

Fundamental wave

2\textsuperscript{nd} harmonic generation

3\textsuperscript{rd} harmonic generation

4\textsuperscript{th} harmonic generation

In copper, Plasma frequency $\sim 1 \, \text{PHz}$, charge relaxation time $\sim 1 \, \text{fs}$
Beam
- Energy = 150 MeV
- Peak current = 3.0 kA
- Energy spread = 1x10^{-3}
- Emittance = 3-mm-mrad
- Initial bunching factor = 0 - 1000 ppm

Undulator
- Period = 5 mm
- Gap = 1.7 mm
- Undulator parameter = 0.4
- Length = 3-6 m

Radiation
- Wavelength = 32.6 nm
Chirped Pulse FEL

Chirped Pulse Compression \( \rightarrow \) high peak power

\[ E(t) \xrightarrow{\text{Positive group velocity dispersion}} e^{j\phi(\omega)} \]

Chirped pulse \( \rightarrow \) Compressed pulse

\[ E(t) = \mathcal{S}^{-1}\left[ E_c(\omega) = E(\omega)e^{j\phi(\omega)} \right] \]

If we send a chirped pulse into a pulse compressor, it emerges unchirped.

This device has **negative group-delay dispersion** and hence can compensate for propagation through materials (i.e., for positive chirp).

http://frog.gatech.edu/pulse-compression.html

Spectral Energy of undulator radiation (fundamental mode)

\[
\left( \frac{d^2W}{d\omega d\Omega} \right) = Q^2 \times B^2(\omega) \frac{N_u^2 \gamma^2}{2\pi} \eta \frac{a_u^2}{(1 + a_u^2 + \gamma^2 \theta^2)^2} \times JJ^2 \left\{ \frac{\sin[N_u \pi(\omega/\omega_r - 1)]}{N_u \pi(\omega/\omega_r - 1)} \right\}^2
\]

where \( JJ = [J_1(\frac{1}{2}\frac{a_u^2}{1 + a_u^2}) - J_0(\frac{1}{2}\frac{a_u^2}{1 + a_u^2})] \) for a planar undulator.

Radiation power for \( a_u \sim 1 \) is \( P_Q \sim 0.6 \times Q^2 \times B^2(\omega) \eta f_r^2 \), where \( f_r \) is the radiation frequency.

For \( Q = 0.3 \text{ nC} \) in 0.15 ps and \( B \sim 0.8 \) @ \( f_r = 1 \text{ THz} \), radiation power \( P_Q \sim 13 \text{ MW} \), which is 0.15% of the beam power of a 4 MeV beam from a photoinjector.

e^{-1} energy depletion \( \Rightarrow \) chirped radiation
The frequency chirped radiation field

\[ E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp \left[ j2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} + j\phi_0 \right] \]

\( N_u \): # of undulator period
\( r_\tau \) = electron transit time / energy depletion time

Amplitude Attenuation

\[ |E(t_n)| = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \]

Pulse Elongation

\[ E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp\left[ j2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} + j\phi_0 \right] \]

\[ t_n = f_{r_0} \times (t - \tau_u) / N_u \quad \text{observation time/unperturbed slippage time} \]

\[ 0 \leq t_n \leq [1 + r_\tau + \frac{1}{3} r_\tau^2] \]

Radiation pulse increased from e\(^{-1}\) energy depletion

\[ N_u \lambda_{r_0} / c = N_u f_{r_0} \]

w/o e\(^{-1}\) energy depletion with e\(^{-1}\) energy depletion

\[ \tau = \frac{L_r}{c}, L_r = N_u \lambda_{r_0} \]
Frequency Chirp

\[ E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp\left[ j2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} + j\phi_0 \right] \]

\[ \phi(t) \]

Radiation phase: defining freq. chirp

(1) Initially \((r_\tau t_n << 1)\)

\[ \phi(t_n) = 2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} \sim 2\pi N_u t_n = 2\pi N_u \frac{t}{N_u \lambda r_0 / c} = \omega r_0 t \]

Radiation frequency is that w/o e\(^{-1}\) energy depletion

(2) For \(r_\tau t_n >> 1\),

Instantaneous frequency

\[ f_r(t) = \frac{1}{2\pi} \frac{d\phi(t_n)}{dt} \sim \frac{N_u}{(3r_\tau t_n)^{2/3}} \quad t_n \to \infty \to 0 \]
$r_\tau = 1.35, N_u = 28$

lengthened pulse due to $e^{-1}$

energy depletion
Design Example

TRW undulator

\[ \lambda_u = 3.56 \text{ cm} \]
\[ a_u = 0.68 \]

UV laser pulse

Solenoid

undulator

\[ 12 \text{ cm} \]
\[ 50 \text{ cm} \]
\[ 100 \text{ cm} \]

4 MeV Photocathode gun

*Photocathode gun – courtesy of CX Tang of Beijing Tsinghua U and W. K. Lau of NSRRC*
Bunch Charge = 200 pC, initial $f_{r0} = 1.1$ THz

30% pump depletion

50% frequency chirp
Dielectric Laser Accelerator

1. Solid state → stable

2. Dielectric damage field and thus high acceleration gradient (up to 1-10 GeV/m)

3. Fabrication compatible to semiconductor lithographic patterning technique

\[ E_{//} \approx \frac{-j}{k} \nabla_E \perp \sim 1 \text{ GV/m} \]

Get your imagination wild….

**Transverse PBG**

\[
\frac{R_{\text{int}}}{\lambda} = 0.68 \quad \Rightarrow \quad \begin{cases} 
Z_{\text{ext}} = 19.5 \, [\Omega] \\
\beta_{\phi} = 0.58 \\
\frac{E_{\text{sec}}}{E_{\text{max}}} = 0.5
\end{cases}
\]

E. Lin, PR STAB, 2000

**Longitudinal PBG**

\[
\frac{D_{\text{int}}}{\lambda} = 0.55 + 1.25 \quad \Rightarrow \quad \begin{cases} 
Z_{\text{ext}} = \frac{\Delta_T}{\lambda} = 250 + 20 \, [\Omega] \\
\beta_{\phi} = 0.2 + 0.6 \\
\frac{E_{\text{sec}}}{E_{\text{max}}} = 0.35 + 0.15
\end{cases}
\]

B. Cowan, PR STAB, 2003

**Cylindrical Bragg Structure**

\[
\frac{R_{\text{int}}}{\lambda} = 0.3 + 0.8 \quad \Rightarrow \quad \begin{cases} 
Z_{\text{ext}} = 268 + 37 \, [\Omega] \\
\beta_{\phi} = 0.41 + 0.48 \\
\frac{E_{\text{sec}}}{E_{\text{max}}} = 0.73 + 0.37
\end{cases}
\]


**Planar Bragg Structure**

\[
\frac{D_{\text{int}}}{\lambda} = 0.3 + 0.8 \quad \Rightarrow \quad \begin{cases} 
Z_{\text{ext}} = \frac{\Delta_T}{\lambda} = 147 + 25.7 \, [\Omega] \\
\beta_{\phi} = 0.42 + 0.53 \\
\frac{E_{\text{sec}}}{E_{\text{max}}} = 0.47 + 0.20
\end{cases}
\]

Demonstration of electron acceleration in a laser-driven dielectric microstructure

Dielectric Laser Accelerator (DLA)

<table>
<thead>
<tr>
<th>μ-bunch length (0.1~1% $\lambda$)</th>
<th>M-pulse length</th>
<th>bunch charge</th>
<th>energy spread</th>
<th>norm. emittance</th>
<th>peak current</th>
<th>M-pulse rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10 nm or 3.3~33 as</td>
<td>~100 fs</td>
<td>$10fC \sim 1pC$</td>
<td>~0.1%</td>
<td>$10^{-9}\sim10^{-11}$ m-rad</td>
<td>0.3~20 kA</td>
<td>~ MHz</td>
</tr>
</tbody>
</table>

$\tau_{M,FWHM} = 100\,fs (\sigma_M \sim 42.5\,fs)$
$\tau_b \sim 2.67\,fs \,(\lambda = 800\,nm)$,
$\tau_\mu = \tau_b / 300 (\sigma_\mu \sim 3.8\,as)$

$I_e(t) = I_0 e^{-t^2/(2\sigma_M^2)} \sum_{m=-\infty}^{\infty} e^{-(t-2m\pi / \omega_b)^2/(2\sigma_\mu^2)}$
Quantum FEL

Pro: quantum noise added to startup power \( P_\omega \), usually small, could assist FEL buildup.

Con: (1) 1 photon from 1 electron \( \rightarrow \) low efficiency  
(2) Electron recoil induced energy spread \(<<\) FEL gain bandwidth

Virtual photon (undulator)

To stay in the gain bandwidth  
\[ \frac{\hbar \omega}{\gamma mc^2} \ll \frac{\Delta \gamma}{\gamma} \sim \rho \]

Define quantum \( \rho \) parameter

\[ \bar{\rho} = \rho (\gamma mc^2 / \hbar \omega) \]

Classic regime (\( \gamma \) large enough)  
\( \bar{\rho} \gg 1 \)

Quantum regime  
\( \bar{\rho} \sim < 1 \)

FEL gain bandwidth \( \sim \Delta \gamma / \gamma \)
Dielectric Laser Undulators
($\lambda_u >> \lambda_{\text{laser}}$ to operate with large $\gamma$)

\[ \lambda_u = \frac{\lambda}{|1/\beta_e - 1/\beta_p|} \]

$\beta_e = v_e / c$: Electron velocity

$\beta_p = v_p / c$: Laser phase velocity

$\lambda_u = 1 \sim 10^3 \mu m, B_u \sim \frac{E_{\text{laser}}}{c} \sim 3.3 T$

for $E_{\text{laser}} \sim 1 \text{ GV/m}$


DLA-driven soft-x-ray FEL (laser undulator $B_u \sim 3\, \text{T}$, $\lambda_r = 1\, \text{nm}$)

Assume rms beam radius $= 100\, \text{nm}$.

$$\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2}$$

$$\bar{\rho} = \rho \left(\frac{\gamma mc}{\hbar k}\right)$$

$$L_g = \lambda_u / 4\pi \sqrt{3} \rho$$

Straight lines are gain-length contours in mm

<table>
<thead>
<tr>
<th>$\bar{\rho}$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1$</td>
<td></td>
</tr>
<tr>
<td>$1 &lt; \bar{\rho} &lt; 10$</td>
<td></td>
</tr>
<tr>
<td>$10 &lt; \bar{\rho} &lt; 100$</td>
<td></td>
</tr>
<tr>
<td>$100 &lt; \bar{\rho}$</td>
<td></td>
</tr>
</tbody>
</table>

$\lambda_u = 20\, \mu\text{m}$

$50\, \text{MeV}$ line

Peak current $= 5\, \text{kA}$, gain length $= 1\, \text{mm}$, $\rho \sim 10^{-3}$

To be published in Review of Modern Physics
DLA-driven hard-x-ray FEL (laser undulator $B_u \sim 3$ T, $\lambda_r = 1$ Å)

Assume rms beam radius = 100 nm

$$\lambda_r = \lambda \frac{1 + a_u^2}{2\gamma^2}$$

$$\bar{\rho} = \rho (\gamma mc / \hbar k)$$

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

Straight lines are gain-length contours in mm

- $\bar{\rho} < 1$
- $1 < \bar{\rho} < 10$
- $10 < \bar{\rho} < 100$
- $100 < \bar{\rho}$

Quantum regime

300 MeV line

Peak current ~ 12 kA, gain length ~ 3 mm

$\lambda_u = 100$ μm

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